Graph Theory Homework 1

Due: 28 May 2019 at 3:59pm as a PDF on Submitty

v1.1: Updated 23 May 2019

1. Construct five pairwise non-isomorphic undirected graphs each satisfying the following conditions:

   (a) the graph has six vertices;

   (b) the graph has eight edges; and

   (c) the graph has a cycle containing all six vertices.

   For each of the graphs, show its degree sequence. Explain why your graphs are not isomorphic to each other.

2. Draw the undirected graph $G = (V, E)$ defined below, labeling vertices and edges, and create its adjacency matrix representation:
   $V = \{v_1, v_2, v_3, v_4, v_5\}$
   $E = \{e_1(v_1, v_2), e_2(v_1, v_5), e_3(v_2, v_3), e_4(v_2, v_4), e_5(v_2, v_5), e_6(v_3, v_5), e_7(v_4, v_5)\}$

   Is G Eulerian? Is G bipartite? How many connected components does G have? Justify each response.

3. Consider a graph $G = (V, E)$, where $|V| \leq |E|$. Use induction to prove that graph G contains a cycle. Note that I didn't specify if G is connected or not.

4. Consider simple connected graph G and its decomposition D. Assume that $|E(G)|$ is even. Show using induction that $\exists D = \{P_2, P_2, \ldots, P_2\}$, where P_2 is path of length 2.

5. In class we saw that if G has no odd cycles $\implies$ G is bipartite. Re-prove this. But this time, use the magical power of induction. You only need to prove the direction of the equivalence relation given above.